AP Calculus AB — Area & Volume

1) $y = 3 - x^2$ — Use symmetry

a) Area $= 2\int_0^1 (3-x^2)\,dx$

$= 2\left[3x - \frac{1}{3}x^3\right]\Big|_0^1$

$= 2\left[3 - \frac{1}{3}\right]$

b) Disc $-dx$ $\quad R = 3-x^2$

Volume $= 2\pi\int_0^1 (3-x^2)^2\,dx$

$= 2\pi\int_0^1 (9 - 6x^2 + x^4)\,dx$

$= 2\pi\left[9x - 2x^3 + \frac{1}{5}x^5 + C\right]_0^1$

$= 2\pi\left[9 - 2 + \frac{1}{5}\right]$

c) WASHER $-dx$

$\pi\int R^2 - \pi\int r^2$

$R = 4 \qquad r = 4 - (3-x^2)$

Volume $= 2\pi\int_0^1 4^2\,dx - 2\pi\int_0^1 \left[4-(3-x^2)\right]^2 dx$

d) semicircles $-\ \frac{\pi}{8}\int d^2$

Volume $= \frac{\pi}{8}\int_{-1}^{1}\left[3-x^2\right]^2 dx$

2) a) $Y_1 = \ln x \qquad Y_2 = 5 - x$

$Y_1 = Y_2$ @ $x = 3.693 \rightarrow A$

Area $= \int_1^A Y_1\,dx + \int_A^5 Y_2\,dx = 2.985$

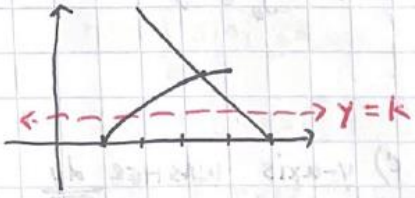
b) Volume $= \int_1^A (Y_1)^2\,dx + \int_A^5 (Y_2)^2\,dx$

$= 2.784$

c) $\int dy$

$y = \ln x \qquad x = e^y$

$y = 5 - x \qquad x + y = 5 \qquad x = 5 - y$



$\int_0^k \left[5 - y - e^y\right] dy = \frac{1}{2}(2.985)$

3) WASHER - dx

$\pi\int R^2dx - \pi\int r^2dx$

$R = x^2 \qquad r = x^3$

$V = \pi\int_0^1 (x^2)^2dx - \pi\int_0^1 (x^3)^2dx$

$= \pi\int_0^1 x^4dx - \pi\int_0^1 x^6dx$

$\pi\left[\frac{1}{5}x^5\right]_0^1 - \pi\left[\frac{1}{7}x^7\right]_0^1$

$\frac{\pi}{5} - \frac{\pi}{7} = \boxed{\frac{2\pi}{35}}$

4) a)

$\text{Area} = \int_0^2 (4-x^2)dx$

$\left[4x - \frac{1}{3}x^3 + C\right]_0^2$

$8 - \frac{8}{3} = \boxed{\frac{16}{3}}$

b) Disc - dx

$V = \pi\int_0^2 (4-x^2)^2dx$

$= \pi\int_0^2 (16 - 8x^2 + x^4)dx$

$= \pi\left[32 - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$

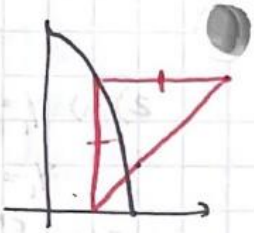
$= \boxed{\pi\left[32 - \frac{64}{3} + \frac{32}{5}\right]}$

c) WASHER - dx

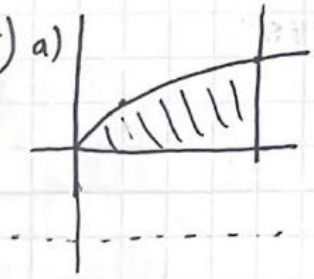
$V = \pi\int_0^2 (6)^2dx - \pi\int_0^2 \left[6-(4-x^2)\right]^2dx$

d) $V = \int \frac{1}{2}(\text{base})(\text{height})$

$V = \int_0^2 \frac{1}{2}(4-x^2)(4-x^2)dx$



5) a)



a) x-axis - Disc dx

$V = \pi\int_0^4 (\sqrt{x})^2$

b) $y = -2$ WASHER dx

$V = \pi\int_0^4 \left(\sqrt{x} - (-2)\right)^2dx - \pi\int_0^4 (2)^2dx$

c) y-axis WASHER dy

$y = \sqrt{x}$

$x = y^2 \qquad V = \pi\int_0^2 [4]^2dy - \pi\int_0^2 [y^2]^2dy$

d) $x = 6$ WASHER dy

$V = \pi\int_0^2 \left[6 - \sqrt{x}\right]^2dy - \pi\int_0^2 [6-4]dy$